The Golden Digits International Contest
7th Edition, December 2024

Problem 1. On a table, there are 2025 empty boxes numbered $1, 2, \ldots, 2025$ and 2025 balls with weights $1, 2, \ldots, 2025$. Starting with Vadim, Vadim and Marian take turns selecting a ball from the table and placing it into an empty box. After all 2025 turns, there is exactly one ball in each box. Denote the weight of the ball in box i by w_i. Marian wins if

$$\sum_{i=1}^{2025} i \cdot w_i \equiv 0 \pmod{23}.$$

If both players play optimally, can Marian guarantee a win?

Problem 2. Let $n \in \mathbb{N}$, $n \geq 2$. Find all functions $f : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ such that

$$f^2(x_1 + \cdots + x_n) = \sum_{i=1}^{n} f^2(x_i) + 2\sum_{i<j} f(x_i x_j),$$

for all $x_1, \ldots, x_n \in \mathbb{R}_{>0}$.

Problem 3. Let ABC be a scalene acute triangle with incenter I and circumcircle Ω. M is the midpoint of small arc BC on Ω and N is the projection of I onto the line passing through the midpoints of AB and AC. A circle ω with center Q is internally tangent to Ω at A, and touches segment BC. If the circle with diameter IM meets Ω again at J, prove that JI bisects $\angle QJN$.

Time allowed: 4 hours and 30 minutes
Each problem is worth 7 points