The Golden Digits International Contest
8th Edition, January 2025

Problem 1. Alex and Bob play a game: Bob picks an initial positive integer x_0. Then, after every minute, Alex chooses a positive integer a, and Bob chooses x_{i+1} to be equal to $x_i + a$ or $x_i + 2a$. Prove that no matter the choice of x_0 and Bob's strategy, Alex can force him to choose a number that is a perfect square after a finite number of minutes.

Problem 2. Let n, m be two integers such that $2 \mid mn$. On an $n \times m$ board we place $\frac{mn}{2}$ dominoes without overlap. On some domino e lies a burrito. Alex sits on the top-left corner of a domino s and is very hungry. He is allowed to make two types of moves:

a) from a vertex of a domino he can move diagonally to the opposite one

b) if he sits on the corner of some domino d he can move to the top-left corner of d

Alex can eat the burrito if he reaches a corner of e. Can Alex satisfy his belly regardless of the choice of m, n, s, and e?

Problem 3. Let $\mathcal{P}$ and $\mathcal{Q}$ be convex polygons with areas $S_{\mathcal{P}}$ and $S_{\mathcal{Q}}$, respectively, such that every vertex of $\mathcal{Q}$ lies inside or on the boundary of $\mathcal{P}$. Prove that there exists a polygon $\mathcal{R}$, similar to $\mathcal{P}$ and with sides parallel to the sides of $\mathcal{P}$, with area $S_{\mathcal{R}}$, such that every vertex of $\mathcal{R}$ lies inside or on the boundary of $\mathcal{Q}$, and

$$S_{\mathcal{R}} \geq \frac{1}{1000} \cdot \frac{S_{\mathcal{Q}}^2}{S_{\mathcal{P}}}.$$

Time allowed: 4 hours and 30 minutes
Each problem is worth 7 points