The Golden Digits International Contest
5th Edition, October 2024

Problem 1. Find all functions $f : \mathbb{Z}_{>}0 \to \mathbb{Z}_{>}0$ with the following properties:

1) For every natural number $n \geq 3$, $\gcd(f(n), n) \neq 1$, where $\gcd(a, b)$ denotes the greatest common divisor of a and b.

2) For every natural number $n \geq 3$, there exists $i_n \in \mathbb{N}$, $1 \leq i_n \leq n-1$, such that $f(n) = f(i_n) + f(n - i_n)$.

Problem 2. Let ABC be a triangle and P a point in its interior. Circle Γ_A is considered such that it is tangent to rays $(PB$ and $(PC$. Define similarly Γ_B and Γ_C. Let $\ell_A \neq PA$ be the other internal tangent of Γ_B and Γ_C. Prove that ℓ_A, ℓ_B and ℓ_C meet at a point.

Problem 3. Let $a_1 < a_2 \cdots < a_n$ be positive integers, with $n \geq 2$. An invisible frog lies on the real line, at a positive integer point. Initially, a hunter chooses a number k, and then, once every minute, he can check if the frog currently lies in one of k points of his choosing, after which the frog goes from its point x to one of the points $x + a_1, x + a_2 \ldots x + a_n$. Based on the values of $a_1, a_2 \ldots a_n$, what is the smallest value of k such that the hunter can guarantee to find the frog within a finite number of minutes, no matter where it initially started?

Time allowed: 4 hours and 30 minutes
Each problem is worth 7 points